Chapter 9

Inequalities

In this course you have focused on determining solutions when two expressions are equal. By using what you know about solving equations, you can now solve linear and exponential equations for a given variable and represent those solutions graphically. However, what if the two expressions are not equal? In this chapter you will learn how to deal with these types of relationships, called inequalities, and will develop ways to represent solutions to inequalities both algebraically and graphically. You will do this for situations involving one variable, two variables, and systems.

In addition, you will extend your ability to work with mathematical sentences by learning how to write mathematical constraints for situations. You will also learn how to work with equations and inequalities involving absolute value.

Chapter Outline

**Section 9.1** You will study how to solve one-variable inequalities and represent the solutions on a number line. You will also study how to solve one-variable absolute value equations and inequalities.

**Section 9.2** You will study how to solve two-variable inequalities and how to represent the solutions of two-variable inequalities on an xy-coordinate graph.

**Section 9.3** In this section, you will apply what you know about systems of equations to determine the solutions to a system of inequalities.
9.1.1 What if the quantities are not equal?
Solving Linear, One-Variable Inequalities

In this course, you have developed a variety of skills to solve different kinds of equations. Now you will apply these equation-solving skills to solve inequalities.

9-1. As a class, create a “human number line” for each of the following mathematical sentences. You will be assigned a number to represent on the number line. When your number makes the equation or inequality true, stand up to show that your number is a solution. If your number does not make the equation or inequality true, remain seated.

a. $x \geq -2$  
   b. $-1 \leq x \leq 4$  
   c. $x = 3$  
   d. $x < -3$

   e. $x > 25$  
   f. $x \leq 1 \text{ or } x \geq 5$  
   g. $-5 < x \leq 0$  
   h. $|x| \geq 2$

9-2. Based on your observations from problem 9-1, discuss the following questions with your class. Be sure to justify your responses.

   a. Compare the solutions to an inequality (like $x \geq -2$) with that of an equation (like $x = 3$). What is different? What causes this to happen?

   b. How many solutions does an inequality such as $x \leq 1$ have?

   c. How are the results of parts (b) and (g) different from the other inequalities? What about the result of part (f)?

9-3. Write an inequality that represents the solutions shown on each number line below.

   a. 

   b. 

   c. 

   d. 

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9-4. SOLUTIONS TO A LINEAR INEQUALITY

With your team, find at least five $x$-values that make the inequality below true.

$$2x - 5 \geq 3$$

a. How many solutions are there?

b. What is the smallest solution for $x$? This point is called a boundary point.

c. What is the significance of the boundary point? What is its relationship with the equation $2x - 5 = 3$? What is its relationship with the inequality $2x - 5 \geq 3$?

d. Write an inequality that represents the solutions for $x$. On a number line, highlight the solutions for $x$. Be ready to share your number line with the class.

9-5. SOLVING LINEAR INEQUALITIES WITH ONE VARIABLE

Analyze the process for solving an inequality, such as $3 - 2x < 1$, by addressing the questions below.

a. Start with the boundary point. How can you quickly solve for this point? Once you have determined your strategy, find the boundary point for $3 - 2x < 1$.

b. Decide if the boundary point is part of the solution to the inequality. If it is part of the solution, indicate this on a number line with a filled circle (point). If it is not a solution, show this by using an open circle as a boundary point.

c. Finally, to determine on which side of the boundary the solutions lie, choose a point to test in the inequality. If the point is a solution, then all points on that side of the boundary are part of the solution. If the point is not a solution, what does that tell you about the solutions? Write your solutions to $3 - 2x < 1$ as an inequality and represent the solutions on a number line.

9-6. With your team, solve the inequality $3x + 1 < 7$. Decide how to represent these solutions on a number line and be prepared to justify your decisions to the class.
METHODS AND MEANINGS

Inequality Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>&lt;</td>
<td>less than</td>
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<tr>
<td>≤</td>
<td>less than or equal to</td>
</tr>
<tr>
<td>&gt;</td>
<td>greater than</td>
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<tr>
<td>≥</td>
<td>greater than or equal to</td>
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</table>

Just as the symbol "=" is used to represent that two quantities are equal in mathematics, the inequality symbols at right are used when describing the relationships between quantities that are not necessarily equal.

When graphing an inequality on a number line, such as \(x \geq -1\), a filled circle (point) indicates that the value is a solution of the inequality, as shown at right.

An open circle indicates that the value is not part of the solution, as in \(x < -3\), as shown at right.

9-7. Solve each of the following inequalities for the given variable. Represent your solutions on a number line.

a. \(2(3p + 1) > -4\)  
b. \(9k - 2 < 3k + 10\)  
c. \(5 - h \geq 4\)

9-8. Identify the mathematical sentences below as always true, sometimes true, or never true.

a. \(-4 \leq 9\)  
b. \(x < 1\)  
c. \(-5 > -2\)

d. \(3x + 5 = 2\)  
e. \(61 = 61\)  
f. \(-6 < -6\)

9-9. Solve each system of equations below.

a. \(2x + y = -7y\)  
   \(y = x + 10\)  
b. \(3x = -5y\)  
   \(6x - 7y = 17\)

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9-10. Write the equation of an exponential function that passes through the points (2, 48) and (5, 750).

9-11. The Battle Creek Bakery makes bread in large batches. Yeast is added to the bread dough to create carbon dioxide gas bubbles in the dough. The tiny gas bubbles give the bread its light texture. They also make the dough increase in size over time, a process called rising. Below is data collected from six batches of dough.

<table>
<thead>
<tr>
<th>Rising Time (min)</th>
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<tr>
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<td>980</td>
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<td>471</td>
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a. Use your calculator to determine the equation of the LSRL. Sketch the scatterplot.

b. Sketch the residual plot and interpret it.

c. Since this equation involves increasing growth over time, try fitting an exponential model to your data. What is the equation of the exponential model that fits the data?

d. Make a residual plot for the exponential model and interpret it.

9-12. For each figure below, determine if the two smaller triangles in each diagram are congruent. If so, write an appropriate triangle congruence statement (you do not have to make a flowchart). Then solve for $x$. If the triangles are not congruent, explain why not.

a. 

b. 

9-13. Rewrite each of the following expressions without parentheses or negative exponents.

a. $(5a^{-2}b^{3})^{8} \cdot (5ab^{-2})^{-6}$

b. \[
\frac{15x^{-5}y^{2}}{(3x^{2})^{2}y^{-3}}
\]
9.1.2 How can I use inequalities?

More Solving Inequalities

In Lesson 9.1.1 you learned how to solve inequalities with one variable. Today you will focus on special inequalities and learn how you can use inequalities to solve an application problem.

9-14. Review what you learned in Lesson 9.1.1 to solve the inequalities below. Represent your solutions both as an inequality and on a number line.

a. \( x - 7 < -2 \)

b. \( 3m + 2 \leq 8m - 8 \)

c. \( \frac{2}{3} p - 2 > -4 \)

d. \( 2 - 3(x - 1) \geq x - 7 \)

e. \( 9k - 4 + 1 \leq 2k - 3 + 7k \)

9-15. THE UNITED NATIONS

At the end of this chapter, your team will have the exciting responsibility of representing a country at a special meeting of the United Nations (U.N.). The U.N. needs your help preparing for future large-scale disasters. You will need to help find a solution that not only works best for the country you represent, but that also accommodates the needs of each of the other countries.

To prepare you for this task, the next several lessons will present daily problems to familiarize you with the important issues and concerns of other countries.

Write and solve an equation that represents the problem below. Be sure to define any variables you use.

There are a total of 193 member states (countries) in the United Nations as of 2014. The member states are divided into regional groups. The Asia group has one less member than the Africa group. The European groups are smaller: Eastern Europe has four less than half the Africa group. The Western Europe group (which includes the United States) has 6 more than Eastern Europe. The Latin America group has 21 less than the Africa group. One country (Kiribati, in the central Pacific Ocean) is in no regional group at all. How many countries are in each of the five groups?
In 1912, Japan gave the United States several thousand flowering cherry trees as a symbol of friendship. Similarly, the nation of Cameroon plans to give flowering Satta trees to other countries this year. When asked how to decide which Satta trees make good gifts, Cameroon’s chief arborist explained:

“We plant Satta trees when they are 6 cm tall, and they grow 9 cm every year. The trees only flower when they are taller than 150 cm.”

It is very important that the trees Cameroon gives flower this year! It would be considered an insult to receive a tree that did not bloom. Luckily, Cameroon has many groves of Satta trees from which to select its gifts. How old must the trees be so that they will flower within the year?

a. Discuss with your team whether an inequality or an equation is appropriate for this situation. Be prepared to share your reasoning.

b. Write and solve a mathematical sentence to determine how old the trees can be so that they flower this year.

c. Later, the arborist added:

“I almost forgot to tell you! When the trees become very old, they stop flowering. Make sure you choose trees that are no more than 240 cm tall!”

Discuss with your team how you can use this additional information to make sure you choose trees that will flower. Be prepared to share your answer with the class.
9-17. Solve the inequalities below for the given variables. Represent your solutions on a number line.
   a. \(3(2k - 1) < 9\)  
   b. \(\frac{2p}{5} \leq 6\)  
   c. \(-2 + 8n > 2\)  
   d. \(7t - 4 > 2t - 4\)

9-18. Thui wrote the following inequalities for a problem that she was solving: \(2n - 1 < 5\) or \(2n - 1 > -5\). Which of the following statements is true?
   a. \(n \geq -3\) or \(n \leq 3\)  
   b. \(n \geq -2\) or \(n < 3\)  
   c. \(n > -3\) or \(n < 3\)  
   d. \(n > -2\) or \(n < 3\)

9-19. Mr. Guo is thinking of a number. When he takes the absolute value of his number, he gets 15. What could his number be? Is there more than one possible answer?

9-20. Use the graph at right to write an equation in \(y = ab^x\) form.

9-21. Turkey has a population of 66 million people and is made up almost entirely of two ethnic groups: Turks and Kurds. There are four times as many Turks as Kurds. How many Kurds live in Turkey?

9-22. Griffin ran 8 miles. Griffin was a football fan so he wondered how many times he had run the equivalent of the full-length of a football field (100 yards). How many times did Griffin run the length of a football field when he ran 8 miles?

9-23. Consider the sequence 2, 8, 3y + 5, …
   a. Find the value of \(y\) if the sequence is arithmetic.
   b. Find the value of \(y\) if the sequence is geometric.
9.1.3 How can I solve the inequality?

Solving Absolute Value Equations and Inequalities

The three approaches you learned in Chapter 3 for solving equations, rewriting, undoing, and looking inside, can also be used to solve inequalities. While the one-variable inequalities you solve today look different than the ones at the beginning of this chapter, the basic process for solving them is similar. As you solve equations and inequalities in today’s lesson, ask yourself these questions:

How can we represent it?

What connections can we make?

9-24. Review what you learned at the beginning of this chapter to solve the inequality $2x + 7 < 12$. Represent the solution algebraically and on a number line.

a. What is the boundary point? Is it part of the solution? Why or why not?

b. In general, how do you determine a boundary point? How do you determine the solutions of an inequality after you have solved for the boundary point? Briefly review the process with your team.

9-25. Absolute value is the numerical value of a number without its sign. Absolute value can represent the distance on a number line between a number and 0.

Using the looking inside strategy for solving equations from Chapter 3, solve $|3x - 5| = 16$. Work with your team to organize your work so that anyone could follow along to find both solutions. Check your solution(s).

9-26. Henry is riding his bike along a straight road when he gets a call from his mom asking where he is. Henry says he is 7 miles from home.

a. Draw a number line with the integers from -10 to 10, representing the road Henry is traveling along. Place Henry’s home at 0. Then mark the position(s) on the road (number line) where Henry could be when his mom called.

b. Use absolute value to write an equation giving Henry’s possible location.
9-27. Henry continues along his route when his friend Emerson calls asking where he is. Henry says “I am 3 miles east of my house.” “Great!” says Emerson, “I am just 2 miles away. I’ll see you soon!” and hangs up.

a. Draw a new number line, marking Henry’s position and Emerson’s possible positions.

b. Write an equation that can be solved to determine where Emerson could possibly be.

c. Solve your equation algebraically. Do your solutions match your number line in part (a)?

9-28. Amanda messaged Henry with the following message: I am still more than 5 miles away. Henry is not sure if Amanda knows that he is not at home.

a. If Amanda thinks Henry is at home, draw a number line and highlight Amanda’s possible positions. Then write an absolute value inequality to represent Amanda’s possible positions on the number line.

b. Maybe Amanda talked to Emerson and knew that Henry was 3 miles east of his house. Draw a number line and highlight Amanda’s possible positions for this situation. Then write the corresponding absolute value inequality.

c. Solve your inequalities in parts (a) and (b) algebraically. Do your algebraic solutions match your number lines?

9-29. Henry is stopped 3 miles east of his house and knows that by now Emerson must be less than 2 miles away. Write and solve an absolute value inequality for Emerson’s possible positions. Graph your solutions on a number line. Do they make sense in the context of this situation?

9-30. Now that you have practiced writing and solving equations and inequalities with absolute value, use your knowledge from the beginning of this chapter to consider the inequality \( |x - 2| > 3 \).

a. Can you use the process from problem 9-24 to solve this inequality? How is it different from solving \( |x - 2| = 3 \)? Solve the inequality and represent your solution algebraically and on a number line.

b. How was solving \( |x - 2| > 3 \) different from solving an inequality without absolute value, such as \( 2x + 7 < 12 \)?
9-31. Solve the inequalities below, if possible, and represent your solution on a number line.

a. \(|x + 2| > 1\)

b. \(|2(x - 1)| \geq 0\)

c. \(9x - 4 \leq 6 - x\)

d. \(|3x - 11| < -2\)

9-32. LEARNING LOG

In your Learning Log, explain how you can solve an inequality that has an absolute value. Then make up your own example problem and show how that problem is solved. Title this entry “Solving Equations and Inequalities with Absolute Value” and include today’s date.
To solve a one-variable linear inequality, first treat the problem as if it were an equality. The solution to the equality is called the boundary point. For example, \( x = 12 \) is the boundary point for the inequality \( 10 - 2(x - 3) \geq -8 \), as shown below.

Problem: \( 10 - 2(x - 3) \geq -8 \)  

First change the problem to an equality and solve for \( x \):

\[
\begin{align*}
10 - 2(x - 3) &= -8 \\
10 - 2x + 6 &= -8 \\
-2x + 16 &= -8 \\
-2x &= -24 \\
x &= 12
\end{align*}
\]

Since the original inequality is true when \( x = 12 \), place your boundary point on the number line as a solid point. Then test one value on either side of the boundary point in the original inequality to determine which set of numbers makes the inequality true. Highlight the points on the side on the boundary point the make the inequality true.

Therefore, the solution is \( x \leq 12 \).

When the inequality is < or >, the boundary point is \textit{not} included in the answer. On a number line, this is indicated with an open circle at the boundary point.
9-33. Solve each equation. Be sure to find all possible solutions and check your solutions.
   a. $|x| = 7$
   b. $|2x| = 32$
   c. $|x + 7| = 10$
   d. $5|x| = 35$

9-34. Determine if the following statements are true or false.
   a. $|-6| < 4$
   b. $|-3 + 5| > 2.5$
   c. $4 \geq |0|$
   d. $|-4 + 3| > 1$

9-35. Solve the following systems of equations.
   a. 
   \[3x - 2y = 14\]
   \[-2x + 2y = -10\]
   b. 
   \[y = 5x + 3\]
   \[-2x - 4y = 10\]
   c. Which system above is most efficiently solved by using the Substitution Method? Explain.
   d. Which system above is most efficiently solved by using the Elimination Method? Explain.

9-36. Write the equation of an exponential function of the form \(y = ab^x\) passing through the points \((2, 3)\) and \((5, \frac{1}{2})\).

9-37. The increased demand for vegetarian meals has caused an increase in the price of tofu. If the cost of tofu is currently $2.99 per pound, and is increasing by 6% per year, what will it cost in 5 years?

9-38. On graph paper, plot and connect the points to form quadrilateral \(QUOP\) if its vertices are \(Q(5, 4)\), \(U(2, 4)\), \(O(-1, 10)\), and \(P(6, 10)\).
   a. Calculate the perimeter of \(QUOP\). Write your answer with appropriate precision.
   b. Rotate quadrilateral \(QUOP\) 90° clockwise (\(\bigcirc\)) about the origin to form quadrilateral \(Q'U'O'O'\). What is the slope of \(Q'P'\)?
A gallon of milk that cost $3.89 a year ago now costs $4.05.

a. If the cost is increasing linearly, what is the growth rate? If the cost kept increasing in the same way, what will the milk cost 5 years from now?

b. If the cost is increasing exponentially, what is the growth rate? What will the milk cost in 5 years?

9.2.1 What if the inequality has two variables?

Graphing Two-Variable Inequalities

In Section 9.1 you learned how to use an inequality with one variable to help solve a word problem. You also discovered that a one-variable inequality can have infinitely many solutions and that these solutions can be represented on a number line. But what if an inequality has two variables? What is a solution to a two-variable inequality? How could these solutions be represented graphically?

9-40. EXAMINING THE SOLUTIONS OF A LINEAR EQUATION

Graph \( y = -2x + 3 \). Compare your graph with the poster graph provided by your teacher.

a. Is the point \((-1, 5)\) a solution to the equation \( y = -2x + 3 \)? How can you tell by looking at the graph? How can you tell by using the equation?

b. Is the point \((2, -1)\) a solution? What about the point \((0, 0)\)? Justify each conclusion using both the graph and the equation.

c. What determines if a point lies on the line? What is the difference between the points on the line and the points not on the line?
9-41. **GRAPHING A LINEAR, TWO-VARIABLE INEQUALITY**

In problem 9-40, you found that the points on the line are the *only* points that make the equation \( y = -2x + 3 \) true. But what if you want to graph the solutions for the inequality \( y \geq -2x + 3 \)? How will that graph differ from the graph of \( y = -2x + 3 \)? Consider this question as you follow the steps below.

a. Your team will be given a list of points to test in the inequality \( y \geq -2x + 3 \). For each point that makes the inequality true, place a sticky dot on that point on the class graph.

b. Now examine the solutions shown on the graph. With your team, discuss the questions below. Be ready to share your conclusions with the class.
   - Are there any points on the graph that you suspect are solutions but do not have a sticker?
   - Are there any stickers that you think may be misplaced? If so, verify these points so that you can have a complete graph of the solutions.
   - What about the points on the line? Are they all solutions to the inequality \( y \geq -2x + 3 \)? Why or why not?
   - How many solutions are there?
   - Where are all of the solutions located relative to the line? Why are there no solutions on the other side of the line?

9-42. What else can you learn about solutions of linear inequalities? Think about this as you answer the questions below with your team.

a. Rather than placing individual sticky dots, what if an entire region of the graph were shaded like the one at right? What inequality would correspond with this graph?

b. Heidi asks, "*What if I changed the inequality to be \( y < -2x + 3 \)? Now what would the graph look like?*" Discuss this with your team and decide the best way to represent the solutions to the inequality \( y < -2x + 3 \). Be prepared to share your graph with the class.
9-43. Graph the inequalities below on graph paper. For each inequality:

- Graph the boundary as either a solid or a dashed line.
- Shade the region that makes the inequality true.

a. \( y > -\frac{1}{3}x - 1 \)  
   b. \( y \leq 4x + 2 \)

c. \( y < \frac{5}{2}x + 3 \)  
   d. \( 2x - y \leq 5 \)

9-44. **LEARNING LOG**

In your Learning Log, explain how to graph a linear inequality. Be sure to address the questions below. Title this entry “Graphing Linear Two-Variable Inequalities” and include today’s date.

- How can you determine if the line is part of the solution?
- How can you determine which region the solution belongs to?
- What point(s) is (are) easiest to test?
- How many points do you need to test?

9-45. Match each graph below with the correct inequality.

a. \( y > -x + 2 \)  
   b. \( y < 2x - 3 \)  
   c. \( y \geq \frac{1}{2}x \)  
   d. \( y \leq -\frac{4}{3}x + 2 \)

1.  
2.  
3.  
4.
9-46. Solve each inequality. Represent the solutions on a number line.

a. \( 3x - 2 < 10 \)  
b. \( 5x - 1 - 3x \geq 4x + 5 \)  
c. \( 2(x + 2) > 10 - x \)  
d. \( 4(x - 3) + 5 \geq -7 \)

9-47. Write and solve an absolute value inequality for this situation.

At Acme Trucking, the average starting annual salary is $24,000 but the actual salary could differ from the average by as much as $1575. What is the range of salaries?

9-48. Use the graph at right to write an exponential equation in \( f(x) = ab^x \) form.

9-49. A line has intercepts \((4, 0)\) and \((0, -3)\). Write the equation of the line.

9-50. Decide if the pair of triangles is congruent. If it is, create a flowchart that justifies your decision.

9-51. Coryn and Leah are working on homework problems that involve arithmetic sequences. Coryn wrote down \((2, 4)\) and \((3, 5)\) from one sequence, and Leah had \((3, 2)\) and \((0, 3)\) for a different sequence. Write an equation for each sequence.
9.2.2 What if the inequality is not linear?

Graphing Linear and Nonlinear Inequalities

In Lesson 9.2.1 you discovered that the solutions of a linear inequality with two variables can be represented by a shaded region on one side of a line. But how can the graph of an inequality help solve an everyday problem? And what happens when the inequality is not linear? Consider these questions as you complete the following problems with your team.

9-52. Review what you learned about graphing inequalities in Lesson 9.2.1 by graphing the inequality below on graph paper.

\[ y \geq -\frac{5}{3} x - 3 \]

a. What is the minimum number of points you need to test in order to know which region contains the solutions?

b. Orville thinks that using the point (0, 0) to test this inequality is a great idea. Why is using this point so convenient?

c. Anita decided to use the point (-3, 2) to test the inequality. Test the inequality with her point. Does this point help her decide which side to shade? Why or why not?

9-53. FOREIGN AID

One of the purposes of the United Nations is to have nations work together to help each other. Recently, the members of the U.N. decided to give grants to poor countries to help reduce poverty. However, the United Nations only has the resources to help those countries with the greatest need. Therefore, it was decided that only countries in which the number of people in poverty is more than one-half of its total population would receive foreign aid.

a. A constraint is a limitation or a restriction. Write an inequality that represents the constraints on receiving foreign aid. Let \( x \) represent the population and \( y \) represent the number of people in poverty.

b. On the Lesson 9.2.2 Resource Page, find the graph that shows the number of people in poverty per the population for each of the countries being considered for foreign aid. Carefully graph your inequality from part (a) on this data graph. Which countries should receive foreign aid?
9-54. What if an inequality is nonlinear? Decide with your team how to graph the inequality $y > 3(0.8)^x$ on graph paper. Are there constraints on your graph besides the given inequality? Explain.

9-55. With your team, graph the following inequalities. Use an $xy$-coordinate graph for all three parts.
   a. $y < 4(1.15)^x$
   b. $y \geq -3$
   c. $x < 2$

9-56. Write an inequality for the solution graphed at right. Be prepared to explain how you decided on your inequality.

9-57. Graph the inequality $x^2 + 3 < x^2 - y + 4 + x$. Be ready to share your graph with the class.
To solve an equation with an absolute value algebraically, first "isolate" the absolute value on one side of the equation.

\[ 5|2x + 3| - 6 = 29 \]
\[ 5|2x + 3| = 35 \]
\[ |2x + 3| = \frac{35}{5} \]
\[ |2x + 3| = 7 \]

Determine the possible values of the quantity inside the absolute value.

For example, if \(|2x + 3| = 7\), then the quantity \((2x + 3)\) must equal 7 or -7.

With these two values, set up new equations and solve as shown below.

\[ |2x + 3| = 7 \]
\[ 2x + 3 = 7 \] or \[ 2x + 3 = -7 \]
\[ 2x = 4 \] or \[ 2x = -10 \]
\[ x = 2 \] or \[ x = -5 \]

Note that distributing over an absolute value is not allowed. For example:

\[ -2|3 + 1| \neq -2(3) + -2(1) \]

9-58. Graph the inequalities below on graph paper.

a. \( y \leq -x + 5 \)

b. \( y > \frac{2}{3} x - 1 \)

9-59. Solve each equation. Be sure to determine and check all of the solutions.

a. \(|9 + 3x| = 39 \)

b. \(|-3x + 9| = 10 \)

c. \(|x + 3| = -2 \)
9-60. Algeria has decided to take out an advertisement in the U.N newspaper, *Liberty Daily*. The newspaper charges a base fee of $1200 for an ad. There is an additional fee of $300 for every inch in height. If Algeria is willing to spend any amount up to (and including) $2700, what choices does the country have for the height of the ad?

9-61. If the graph of an exponential function passes through the points (1, 6) and (4, 48), write an equation for the function.

9-62. Each team in Ms. Zaleski’s biology class is growing oyster mushrooms. The weight of several specimens was recorded along with the number of days of growth. The data that was collected is shown in the table below.

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<th></th>
<th></th>
<th></th>
<th></th>
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<tr>
<td>Weight (g)</td>
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<td>4.6</td>
<td>3.4</td>
<td>2.3</td>
<td>1.6</td>
<td>0.9</td>
<td>0.8</td>
<td>0.3</td>
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<td>1.3</td>
<td>0.1</td>
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</tbody>
</table>

a. Make a scatterplot for the data and sketch it onto your paper. Use time as the independent variable. Describe the association.

b. The weight of the mushrooms depends on growth over time. What kind of equation do you suggest to model this data?

c. Model the data with a regression equation. Does the y-intercept of your model make sense in this context?

d. What do you predict an 8-day-old oyster mushroom will weigh? Use appropriate precision in your answer.

9-63. On a coordinate grid, graph \( \triangle ABC \) if \( A(-3, -4) \), \( B(-1, -6) \), and \( C(-5, -8) \).

a. What is \( AB \) (the length of \( \overline{AB} \))? 

b. Reflect \( \triangle ABC \) across the x-axis to form \( \triangle A'B'C' \). What are the coordinates of \( B' \)? Describe the function that would change the coordinates of \( \triangle ABC \) to \( \triangle A'B'C' \).

c. Rotate \( \triangle A'B'C' \) 90° clockwise (\( \cup \)) about the origin to form \( \triangle A''B''C'' \). What are the coordinates of \( C'' \)?

d. Translate \( \triangle ABC \) so that \((x, y) \rightarrow (x + 8, y + 6) \). Describe the translation as a movement a certain distance along a line parallel to the line of translation.
9.64. Line L is perpendicular to the line \( 6x - y = 7 \) and passes through the point \((0, 6)\). Line M is parallel to the line \( y = \frac{2}{3}x - 4 \) and passes through the point \((-3, -1)\). Where do lines L and M intersect? Explain how you found your solution.

9.3.1 How can I represent it?

Systems of Inequalities

In Chapter 6 you learned that the solution to a system of equations is a point that makes both equations true. But what about the solution of a system of inequalities? How can you represent these solutions on a graph? How many solutions can a system of inequalities have?

Consider these questions as you learn how to graph a system of inequalities.


a. Compare your solution graphs for \( y \leq -x + 5 \) and \( y > \frac{2}{3}x - 1 \) with those of your teammates. Correct any errors. Be sure to focus on whether the boundary line should be included in each graph.

b. What would the graph of a system of inequalities look like? Consider the system of inequalities below. Which points are solutions to this system (that is, which points make both inequalities true)?

\[
\begin{align*}
y & \leq -x + 5 \\
y & > \frac{2}{3}x - 1
\end{align*}
\]

c. If you have not done so already, verify your solution region from part (b) algebraically by substituting the coordinates of a point from your solution region into each inequality.

d. How can you be sure that this region contains the only set of points that makes both inequalities true?
9-66. Draw a graph of the region satisfying both inequalities below. Start by graphing the boundary lines and then test points to determine the region that makes both inequalities true.

\[ y < x + 2 \]
\[ y \leq 10 - \frac{3}{4}x \]

9-67. **HOW MANY REGIONS?**

When graphing the system of inequalities below, Reyna started with the boundary graph of each constraint shown at right.

\[ y \leq 0.5^x - 6 \]
\[ y > -2x \]

a. Why is the line dashed while the exponential function is not?

b. Find a copy of Reyna’s graph on the Lesson 9.3.1B Resource Page. How many possible solution regions are there? Carefully count each region with your teammates.

c. Pick a point in each region and test it in the system of inequalities. Shade any regions that contain solutions to both inequalities. How many regions make up the solution to this system?

d. Why is (0, 0) not a good point to use to test for this solution?

9-68. How does changing the inequality symbol affect the solution graph? Notice that each system of inequalities below uses the same boundary graphs as Reyna’s graph from problem 9-67. However, notice that this time the constraints are slightly altered.

With your teammates, devise a method to determine which region (or regions) are solutions for each system. Shade the appropriate regions on your resource page.

a. \[ y \geq 0.5^x - 6 \]
\[ y > -2x \]

b. \[ y \geq 0.5^x - 6 \]
\[ y < -2x \]

c. \[ y \leq 0.5^x - 6 \]
\[ y < -2x \]
The United Nations asked every nation to write constraints that best approximate its country's shape (the U.N. thinks this will help find each country's area). Honduras sent in its constraints, but some of the information is unreadable. With your team, determine the missing parts of the inequalities and rewrite them on your paper.

\[
\begin{align*}
&\quad y \geq x + 3 \\
&\quad y \geq \frac{1}{2} x - (\text{unreadable}) \\
&\quad y \geq -\frac{2}{3} x + 4 \\
&\quad y \geq -\frac{2}{3} x - 1
\end{align*}
\]
To represent the solutions to an inequality with two variables, first graph the boundary line or curve. If the inequality does not include an equality (that is, if it is \( > \) or \( < \) rather than \( \geq \) or \( \leq \)), then the graph of the boundary is dashed to indicate that it is not included in the solution. Otherwise, the boundary is a solid line or curve.

Once the boundary is graphed, choose a point that does not lie on the boundary to test in the inequality. If that point makes the inequality true, then the entire region in which that point lies is a solution. Examine the two examples below. There are infinitely many solutions to each of the inequalities. The shaded region of the graph is a diagram of all the solutions.

For the given inequality \( y < -\frac{2}{3}x + 2 \), first graph the equation \( y = -\frac{2}{3}x + 2 \) with a dashed line. Then, test the coordinates of a point that is not on the line in the inequality.

- Test \((0,0)\):
  - \( 0 < -\frac{2}{3}(0) + 2 \)
  - \( 0 < 0 + 2 \)
  - \( 0 < 2 \) ✓

Since the inequality is true, shade the region where \((0,0)\) is located. See solution graph at right.

9-70. Graph the system of inequalities below.

\[
\begin{align*}
 y & \geq 2x - 4 \\
 y & < x
\end{align*}
\]

a. Shade the solution region.

b. Is \((0,0)\) a solution to this system? How can you tell?
9-71. **Multiple Choice:** Which of the points below is a solution to \( y < |x - 3| \)?

a. \((2, 1)\)  
b. \((-4, 5)\)  
c. \((-2, 8)\)  
d. \((0, 3)\)

9-72. Write and solve an inequality for this situation.

To honor 50 years in business, All Strikes Bowling is having an anniversary special. Shoes rent for $1.25 and each game is $0.75. If Charlie has $20 and needs to rent shoes, how many games can he bowl?

9-73. Solve the system of equations at right algebraically.

\[
\begin{align*}
2x - 3y &= 12 \\
y + x &= -9
\end{align*}
\]

9-74. Three years ago the average price of a movie ticket was $8.75 and now it is $10.87. What are the annual multiplier and the percent increase? Sketch a graph and make a table for the situation.

9-75. Solve the following equations for \(x\).

a. \(4x - 6y = 20\)  
b. \(\frac{1}{2}(x - 6) = 9\)  
c. \(\frac{4}{3} + \frac{18}{x} = 8\)  
d. \(2 + |2x - 3| = 5\)

9-76. Determine if the triangles at right are congruent. If they are, justify your answer using one of the triangle congruence conditions, and state a sequence of rigid transformations that maps one triangle onto the other.
9.3.2 How can I apply it?

More Systems of Inequalities

In this lesson, you will continue to investigate systems of inequalities as you work to solve a real-world problem with your team.

9-77. Review what you learned about systems of inequalities in Lesson 9.3.1 by graphing the system of inequalities right on graph paper. Carefully shade the region of points that make both inequalities true.

\[ y > 2^x \]
\[ x + 4y \leq 4 \]

9-78. SEARCH AND RESCUE

"I'm completely lost... water everywhere I can see... both burners have failed... Wait! I see land. I'm going to try to land. I think it's..."

Those were the last words heard from Harold in his hot-air balloon. The last time the balloon showed up on radar, it was near the Solomon Islands in the Pacific Ocean.

**Your Task:** Your team must determine where to send the search-and-rescue teams! Use the following reports along with the map on the Lesson 9.3.2 Resource Page and look carefully for information that will help you draw constraints on where the balloon might be found. Give the search constraints to the search-and-rescue team as a system of inequalities. Be sure to identify the probable landing site on the map.

*Problem continues on next page -*
Basic facts of the case:
The balloon departed from the airport at the very northern tip of the Philippines. The flight was supposed to follow a straight path directly to an airport in French Polynesia.

The balloon's last known location was at (−1000, 1000) near the Solomon Islands.

Pilot's report from a nearby airplane:
"We were on our way from Australia, when we saw a hot-air balloon sinking rapidly. I am certain that it crashed south of our flight path. When we left Australia, we traveled 2000 km north for every 3000 km east that we flew."

Phone call received today:
"I was a passenger on a flight that flew directly from French Polynesia to Indonesia. I was looking out my window when I saw the balloon going down to the north of where we were flying."

9-79. Notice that each system of inequalities below contains the same boundary lines. On graph paper, graph the boundaries for the system on one set of axes. Then, for each pair of inequalities, work with your team to decide which region is the solution, if a solution exists. Be ready to share your conclusions with the class.

a. \[ y \leq \frac{2}{3}x + 3 \]
\[ y \geq \frac{2}{3}x \]

b. \[ y \leq \frac{2}{3}x + 3 \]
\[ y \leq \frac{2}{3}x \]

c. \[ y \geq \frac{2}{3}x + 3 \]
\[ y \leq \frac{2}{3}x \]

9-80. LEARNING LOG

In your Learning Log, describe your method for graphing systems of inequalities for a student who has missed class for the last couple of days. Be sure to include examples and important details. Title this entry "Graphing Systems of Inequalities" and include today’s date.

9-81. Graph and shade the solution for the system of inequalities below.
\[ y \geq \frac{3}{4}x - 2 \]
\[ y < -\frac{1}{2}x + 3 \]
9-82. Solve the inequalities below. Graph your solutions on a number line.
   a. \(|b - 4| < 36\)           b. \(|3 + x| - 9 \geq 21\)

9-83. The town you live in has decided to limit the amount of trash thrown out each month. Your town, which has 3280 homes, has asked each household to keep track of how many pounds of trash they produce during a month. In addition, the town council has found that other sources of trash, such as local businesses, combine to create 1500 lbs of trash each month. If the town has a goal of creating less than 50,000 lbs of trash, how much trash should a household be limited to? Write an inequality for this situation and solve it.

9-84. Write an exponential equation to represent the situation and answer the question.

   If the cost of a loaf of bread is now $2.75 and is increasing at 5% per year, what will it cost 10 years from now?

9-85. Clifford thinks that \(x = 7\) is a solution to \(3(x - 2) \leq 4\). Is he correct? Show why or why not.

9-86. During a race, Bernie ran 9 meters every 4 seconds, while Wendel ran 2 meters every second and got a 9-meter head start. If the race was 70 meters long, did Bernie ever catch up with Wendel? If so, when? Justify your answer.

9-87. For each table below, calculate the missing entries and write an equation.
   a. 

   \[
   \begin{array}{l|l|l|l|l|l|l|l}
   \hline
   \text{Month (x)} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
   \hline
   \text{Population (f(x))} & 2 & 8 & 32 & & & & \\
   \hline
   \end{array}
   \]

   b. 

   \[
   \begin{array}{l|l|l|l|l|l|l|l}
   \hline
   \text{Year (x)} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
   \hline
   \text{Population (f(x))} & 5 & 6 & 7.2 & & & & \\
   \hline
   \end{array}
   \]

   c. Which function is growing at a faster rate, the one in part (a) or part (b)? How can you tell from the tables? How can you tell by looking at the equations?
9.3.3 How can I use inequalities to solve problems?

Applying Inequalities to Solve Problems

Today you will pull together all the mathematics you have studied in this chapter and apply it to solve an application problem.

9-88. UNITED NATIONS TO THE RESCUE

As a representative of your country, you have been sent the following letter and given an important task:

Dear Representative to the United Nations:

A critical matter has come to the attention of the United Nations. In the past, when a catastrophe struck a part of the world, the U.N. gathered supplies to give to people in need. Unfortunately, because the U.N. had to collect supplies from each country at the time of the catastrophe, it was always quite a few days before the supplies could be sent to the areas that needed them the most.

A recommendation has come before the U.N. to create a supply of food and medicine packages for future emergencies. Each food package will be able to feed several hundred people, while each medicine package will supply one first-aid station. I am asking each country to donate the same number of packages so each country shares the burden equally.

I am asking each country to determine how many food and medicine packages they are able to give. You will present your findings at today’s United Nations meeting. Please be certain to use the information that your country’s Budget Committee has prepared to help you decide how many packages you can afford.

Best of luck, and may our efforts make our world a better place!

Sincerely,
The Secretary General of the United Nations

After consulting with your country’s Budget Committee, your teacher will supply you with some information that will help you decide how many food and medicine packets your country can afford.

Problem continues on next page →
9-88. *Problem continued from previous page.*

**Your Task:** To communicate your country’s budget constraints, write an inequality expressing how many food and medicine packages your country is able to give. Let \( x \) equal the number of food packages and let \( y \) equal the number of medicine packages.

On the Lesson 9.3.3B Resource Page, graph the solution region representing the number of medicine and food packets that can be donated by your country. Be prepared to share your graph with the other countries of the United Nations.

9-89. As a member of the United Nations, you must consider each of the following proposals. In each case, assume that the United Nations would like to receive as many emergency supplies as possible, while still having each nation give equally.

a. One proposal is that each country gives 185 medicine packages. How many food packages should the United Nations require from each country in this case? Explain how you made your decision.

b. Another proposal is to get the largest number of medicine packages possible. What is the largest number of medicine packages that each country can offer? How did you find your answer?

9-90. A last-minute proposal suggests balancing the number of food and medicine supplies. For instance, if a country gives 150 food packages, then they would also give 150 medicine packages. How many food and medicine packages should the United Nations require from each country in this case? Explain how you determined your solution.
9-91. For the Spring Festival, the Math Club is selling rulers for $1 and compasses for $2.50.

a. While the club would like to sell as many items as they can to raise funds, they need to make at least $15.00 to break even. Write an inequality to represent this situation.

b. School rules state that the club can sell a maximum of 25 items for the festival. Write an inequality for this constraint.

c. Graph the inequalities from parts (a) and (b) on the same set of axes so that compasses are represented on the x-axis and rulers are represented on the y-axis. Find the region of points that are solutions to each of them. Can this region fall below the x-axis or to the left of the y-axis? Why or why not?

d. What do the points in the solution region represent? Are all points in the solution region possible combinations of sales of rulers and compasses?

9-92. Write an inequality that represents the x-values highlighted on each number line below.

a. \[ -10 \quad -9 \quad -8 \quad -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad x \]

b. \[ -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad x \]

c. \[ -10 \quad -9 \quad -8 \quad -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad x \]

d. \[ -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad x \]

9-93. Solve each equation.

a. \[ 3(x - 2) = -6 \]

b. \[ (x + 2)(x + 3) = (x + 1)(x + 5) \]

c. \[ |2x - 5| = 17 \]

d. \[ \frac{2x}{9} = \frac{14}{5} \]

9-94. At the Keaveny Hardware Manufacturing Company they need to meet very specific criteria for the diameter of the threads of #10 wood screws. The diameter of the threads must be \( \frac{\frac{3}{16}}{2} \) of an inch. The width cannot vary more than \( \frac{\frac{1}{64}}{2} \) of an inch from the specified diameter. Write and solve an appropriate absolute value inequality for this situation. Graph your solution.
9.95. This problem is a checkpoint for solving linear systems of equations. It will be referred to as Checkpoint 9.

Solve each system of equations.

a. \[ \begin{align*}
    y &= 3x + 11 \\
    x + y &= 3
\end{align*} \]

b. \[ \begin{align*}
    y &= 2x + 3 \\
    x - y &= -4
\end{align*} \]

c. \[ \begin{align*}
    x + 2y &= 16 \\
    x + y &= 2
\end{align*} \]

d. \[ \begin{align*}
    2x + 3y &= 10 \\
    3x - 4y &= -2
\end{align*} \]

Check your answers by referring to the Checkpoint 9 materials located at the back of your book.

If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 9 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

9.96. Graph \( f(x) = -0.75(2)^x \).

9.97. A sequence starts \(-3, 1, 5, 9, \ldots\)

a. If you wanted to find the 50th term of the sequence, would an explicit equation or a recursive equation be more useful?

b. Write the equation in standard form as you did in problem 8-98.

c. What is the 50th term of the sequence?
Chapter 9 Closure  What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics that you need more help with. Look for connections between ideas as well as connections to material you learned previously.

① TEAM BRAINSTORM

What have you studied in this chapter? What ideas were important in what you learned? With your team, brainstorm a list. Add as much detail as you can. To help get you started, Learning Log entries and Math Notes boxes are listed below.

What topics, ideas, and words that you learned before this chapter are connected to the new ideas in this chapter? Again, write down as many details as you can.

How long can you make your list? Challenge yourselves. Be prepared to share your team’s ideas with the class.

Learning Log Entries

- Lesson 9.1.3 – Solving Equations and Inequalities with Absolute Value
- Lesson 9.2.1 – Graphing Linear Two-Variable Inequalities
- Lesson 9.3.2 – Graphing Systems of Inequalities

Math Notes

- Lesson 9.1.1 – Inequality Symbols
- Lesson 9.1.3 – Solving One-Variable Linear Inequalities
- Lesson 9.2.2 – Solving Absolute Value Equations
- Lesson 9.3.1 – Solutions to Two-Variable Inequalities
MAKING CONNECTIONS

Below is a list of the vocabulary words used in this chapter. Make sure that you are familiar with all of these terms and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

- absolute value
- boundary line
- boundary point
- constraint
- coordinates
- equation
- graph
- greater than
- inequality
- inequality symbols
- less than
- number line
- one variable
- region
- solution
- system of inequalities
- two variable

Make a concept map showing all the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection, as shown in the model below. A word can be connected to any other word as long as you can justify the connection. For each key word or idea, provide an example or sketch that shows the idea.

![Concept Map Example]

Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all the connections explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.
PORTFOLIO: EVIDENCE OF MATHEMATICAL PROFICIENCY

By solving the United Nations to the Rescue problem (9-88), you learned how to solve a challenging systems of inequalities problem, modifying and expanding it if necessary, to showcase your understanding of solving inequalities. Obtain a Chapter 9 Closure Resource Page with a situation for you to solve demonstrating your knowledge of systems of inequalities.

WHAT HAVE I LEARNED?

Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

CL 9-98.  Write an inequality that represents the graph at right.

CL 9-99.  Is the point (0, 4) a solution to the system of inequalities at right?  

Justify your answer.

\[ y \leq -3x + 4 \]
\[ y > 5x - 2 \]

CL 9-100.  Lew says to his granddaughter Audrey, “Even if you tripled your age and added 9, you still wouldn’t be as old as I am.” Lew is 60 years old. Write and solve an inequality to determine the possible ages Audrey could be.

CL 9-101.  The cost to rent a pair of bowling shoes has decreased 10% per year over the past several years. If the current cost is $5, write an equation of an exponential function to model this situation. Then make a table of values and sketch a graph.
CL 9-102. Solve each system of equations.

a. \[2x - y = 9\]
   \[y = x - 7\]

b. \[-4x + y = 5\]
   \[2x = -y - 13\]

CL 9-103. Solve each inequality or equation algebraically. Then represent your solutions on number lines.

a. \[6(x - 2) \geq 12\]

b. \[-3x + 4 < -11\] or \[4x - 5 < -9\]

c. \[|2x - 1| < 5\]

d. \[2|x| - 5 < -1\]

CL 9-104. Write an equation of an exponential function in \[y = ab^x\] form that satisfies each of the following sets of conditions.

a. Has a \(y\)-intercept of \((0, 2)\) and a multiplier of \(0.8\).

b. Passes through the points \((0, 3.5)\) and \((2, 31.5)\).

c. Passes through the points \((3, 13.5)\) and \((5, 30.375)\).

CL 9-105. Write an explicit equation for each of the following sequences.

a. \(100, 10, 1, 0.1, \ldots\)

b. \(0, -50, -100, \ldots\)

CL 9-106. Examine the diagram at right. \(M\) is the midpoint of both \(\overline{KQ}\) and \(\overline{PL}\), and \(m\angle KML = m\angle PMQ\). Demonstrate that the two triangles are congruent by using a flowchart.


a. \(|x| + 3 = 8\)

b. \(|x - 5| = 17\)

CL 9-108. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems made you think? Use the table to make a list of topics you need help on and a list of topics you need to practice more.
Answers and Support for Closure Activity #4  
*What Have I Learned?*

Note: MN = Math Note, LL = Learning Log

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<tr>
<th>Problem</th>
<th>Solution</th>
<th>Need Help?</th>
<th>More Practice</th>
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<td>$y &gt; \frac{1}{2}x - 2$</td>
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<td>Yes, because it makes both inequalities true.</td>
<td>Section 9.3</td>
<td>Problems 9-70 and 9-91</td>
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<td>$3A + 9 &lt; 60$</td>
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<td>$A &lt; 17$</td>
<td>and 9.1.2</td>
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<td>Audrey is less than 17 years old.</td>
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<td>9.1.3</td>
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<td>Section 8.1</td>
<td>Problems</td>
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<td>More Practice</td>
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<tr>
<td>CL 9-103.</td>
<td>a. $x \geq 4$</td>
<td>Lesson 9.1.3</td>
<td>Problems 9-7, 9-17, 9-46, and 9-82</td>
</tr>
<tr>
<td></td>
<td>b. $x &gt; 5$ or $x &lt; -1$</td>
<td>MN: 9.1.3</td>
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<td></td>
<td>c. $-2 &lt; x &lt; 3$</td>
<td>LL: 9.1.3</td>
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<td>d. $-2 &lt; x &lt; 2$</td>
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<tr>
<td>CL 9-104.</td>
<td>a. $y = 2(0.8)^x$</td>
<td>Lesson 8.2.1</td>
<td>Problems 8-96, 8-121, 9-10, 9-36, 9-61, and 9-84</td>
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<td></td>
<td>b. $y = 3.5(3)^x$</td>
<td>MN: 8.2.2</td>
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<td>c. $y = 4(1.5)^x$</td>
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<tr>
<td>CL 9-105.</td>
<td>a. $t(n) = 100(0.1)^{n-1} = 1000(0.1)^n$</td>
<td>Sections 5.1 and 5.2</td>
<td>Problems CL 5-127, CL 6-152, 9-51, and 9-97</td>
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<tr>
<td></td>
<td>b. $t(n) = 0 - 50(n - 1) = -50n + 50$</td>
<td>MN: 5.3.2 and 5.3.3</td>
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<tr>
<td>CL 9-106.</td>
<td>$M$ is a midpoint of $KQ$</td>
<td>Section 7.1</td>
<td>Problems CL 8-135, 9-12, 9-50, and 9-76</td>
</tr>
<tr>
<td></td>
<td>Given</td>
<td>MN: 7.1.2, 7.1.3, 7.1.4, and 7.1.7</td>
<td></td>
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<td></td>
<td>$\angle P \equiv \angle L$</td>
<td>LL: 7.1.2, 7.1.3, and 7.1.5</td>
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<td></td>
<td>$KM \equiv QM$</td>
<td>Definition of midpoint</td>
<td></td>
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<td></td>
<td>$\angle KML \equiv \angle QMP$</td>
<td>Vertical angles are congruent</td>
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<tr>
<td></td>
<td>$\triangle KLM \equiv \triangle QPM$</td>
<td>AAS $\equiv$</td>
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<tr>
<td>CL 9-107.</td>
<td>a. $x = -5$ or 5</td>
<td>Lesson 9.1.3</td>
<td>Problems 9-33, 9-59, and 9-93</td>
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<tr>
<td></td>
<td>b. $x = -12$ or 22</td>
<td>MN: 9.1.3</td>
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<td>LL: 9.1.3</td>
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